TECHNICAL NOTE TN-5

ELEMENTARY MATHEMATICS OF MODEL ROCKET FLIGHT

by Robert L. Cannon
Edited by Ann Grimm

$$v_2 = \left(\frac{F}{w_{av}} - 1\right) gt$$

w_{av} = average weight of rocket

F = force (average thrust of rocket engine)

g = acceleration due to gravity (32 ft./sec.^{2})

$$\text{Force (thrust)} = \frac{\text{Total Impulse}}{\text{Burn Time}} \times \frac{16 \text{ oz.}}{1 \text{ lb.}}$$

$$F = \frac{0.56 \text{ lb.} - \text{sec.}}{0.32 \text{ sec.}} \times \frac{16}{}$$

$= 1.75 \times 16$ oz.

$= 28.0$ oz.

$$v_2 = \left(\frac{F}{w_{av}} -$$

$$v_2 = \left(\frac{28.0 \text{ o}}{1.32}\right.$$

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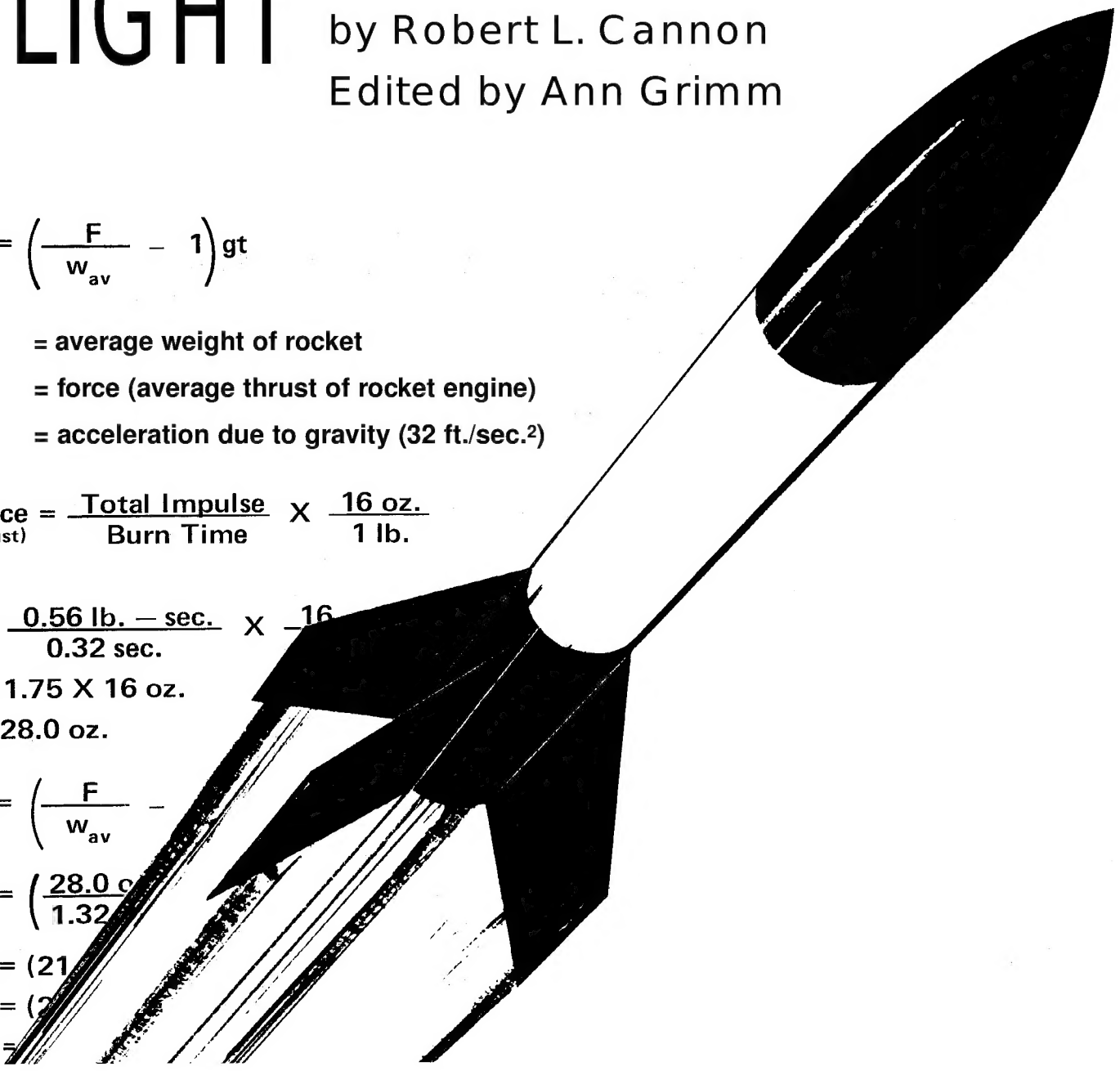
$=$



This publication contains three separate articles on different aspects of model rocket flight.

One article explains how the heights reached by model rockets can be determined. Instructions on building and using your own altitude determining device are included, as well as an elementary explanation of the theory involved.

The second article provides facts and examples for helping the students build better concepts about relative velocities and speeds.

The final article about acceleration provides more advanced information on speed, acceleration and distance traveled per unit of time.

EST #2844

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How High Did It Go??

by ROBERT L. CANNON

"My rocket went higher than your rocket!"

"Did not!"

"Did too!"

"Did not!"

"Did!"

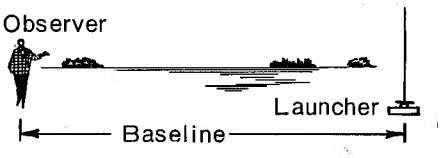
Does this dialogue sound familiar? How often have you heard two *other* rocketeers arguing about whose model went higher? Sheer lung power is not enough to determine whose rocket went the highest.

One of the easiest ways to judge rockets to find which is the "best" is to see which goes the highest with a certain engine type. To be really scientific about determining whose rocket went the highest, even careful "watching" may not be adequate.

Easy Altitude Calculations

Reliable altitude measurements are easy to make. For most purposes, a simple calculation using only three numbers is all it takes to find the altitude of a model rocket.

First, measure to find how far from the launcher you are going to stand when the rocket is launched. If you have a good idea of how high your rocket should go, measure to find a place the same amount of distance from the launcher and stand at this position (baseline). When in doubt about how high a model will go, checking tables of predicted

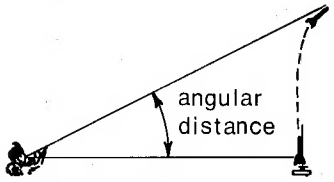


performance, guessing from past experience or using the Estes Altitrak™ are the best methods of predicting the height your rocket will reach.

Measuring this BASELINE can be done with a meter stick (slightly over a yard-39.37 inches), a yardstick if you can't get a meter stick or a metric tape. If you can't get a better measuring device, pace off the distance. To use pacing for measuring the baseline, first measure how far you go each time you take a step, then figure the number of steps it will take to go the necessary distance.

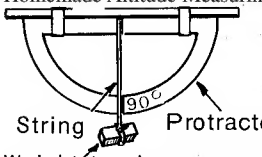
The second number you need is the ANGULAR DISTANCE the rocket travels from launch to apogee (highest point of flight). This is measured in degrees of angle.

The angular distance is determined by



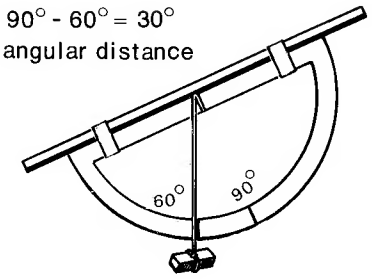
measuring the angle between the rocket's position on the launch pad (the tip of the nose cone) and the highest point (apogee) reached by the rocket.

Homemade Altitude Measuring Device



Sighting tube (can be soda straw)

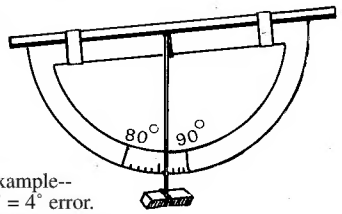
The measuring device used to find this angular distance can be a homemade "sextant" or the Estes Altitrak™.



When using the homemade altitude measuring device, angular distance is found by subtracting the reading taken (angle marked) of the rocket at apogee from 90°.

If you are using a homemade altitude measuring device, a sighting must be made on the tip of the rocket on the launch pad and then the angular error noted (difference between 90° mark and the angle marked by the string). This is an error you will have to allow for when measuring the angular height reached by each flight.

Example



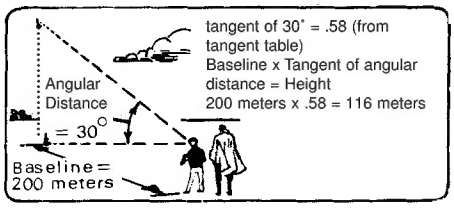
In this example--
90° - 86° = 4° error.
Subtract four degrees from angular distances measured for each flight.

Once the angular distance moved by the rocket is known, consult a trigonometry table to find the TANGENT of that angle.

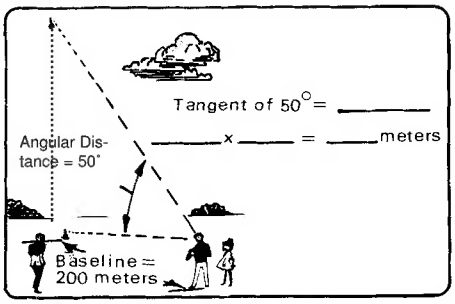
TABLE OF TANGENTS

Angle	Tan.	Angle	Tan.	Angle	Tan.
1°	.02	28°	.53	54°	1.38
2	.03	29	.55	55	1.43
3	.05	30	.58	56	1.48
4	.07	31	.60	57	1.54
5	.09	32	.62	58	1.60
6	.11	33	.65	59	1.66
7	.12	34	.67	60	1.73
8	.14	35	.70	61	1.80
9	.16	36	.73	62	1.88
10	.18	37	.75	63	1.96
11	.19	38	.78	64	2.05
12	.21	39	.81	65	2.14
13	.23	40	.84	66	2.25
14	.25	41	.87	67	2.36
15	.27	42	.90	68	2.48
16	.29	43	.93	69	2.61
17	.31	44	.97	70	2.75
18	.32	45	1.00	71	2.90
19	.34	46	1.04	72	3.08
20	.36	47	1.07	73	3.27
21	.38	48	1.11	74	3.49
22	.40	49	1.15	75	3.73
23	.42	50	1.19	76	4.01
24	.45	51	1.23	77	4.33
25	.47	52	1.28	78	4.70
26	.49	53	1.33	79	5.14
27	.51			80	5.67

The final step to determine the altitude reached is to multiply this value by the length of the baseline. The product is the height reached by the rocket.


tangent of 30° = .58 (from tangent table)
Baseline x Tangent of angular distance = Height
200 meters x .58 = 116 meters
Angular Distance
= 30°
Baseline = 200 meters

Now, calculate the height reached by this rocket.



The rocket rose 238 meters.

Let's determine the height reached by another rocket. The information given is similar to the data given to the data reduction crew at a model rocket contest.

Baseline = 250 meters
Angular Distance = 70°
Altitude reached = _____ meters*

* 687.5 meters

If your calculated altitude was not accurate, recheck your calculations to find and correct your error. Once you are able to correctly find the altitude, find the altitudes for the next two problems to be sure you know how to calculate altitudes.

Baseline = 100 meters
Angular Distance = 53°
Altitude reached = _____ meters*

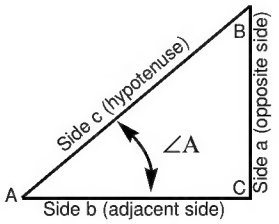
* 133.0 meters

Baseline = 500 meters
Angular Distance = 20°
Altitude reached = _____ meters*

* 180.0 meters

Explanation of Tangents

You may have begun to wonder, "What is this 'tangent' that we've been using?" A tangent is a ratio (a numerical relationship). When working with a *right* triangle (a triangle with one "right" or 90° angle), the tangent is the ratio between the length of the *opposite* side and the length of the *adjacent* or nearest side.

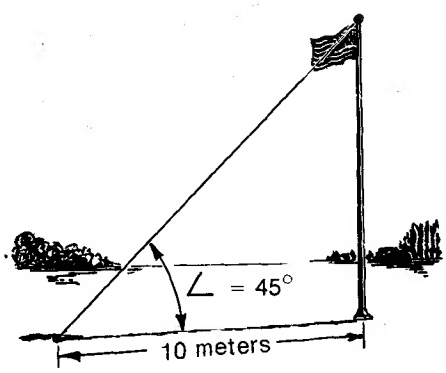


The tangent of angle A is the ratio of the length of opposite *Side a* to the length of the nearest *Side b*. The longest side of a right triangle is always called the *hypotenuse*. The equation form of the tangent of angle A is written:

$$\text{Tangent of } \angle A = \frac{\text{opposite side}}{\text{adjacent side}}$$

Notice how this triangle resembles the situation when you are tracking a rocket. The rocket is launched from C and reaches apogee at B. The length of *Side b* is measured and angle A is measured. To find the length of *Side a* (the height reached by the rocket), we multiply the length of *Side b* (baseline) times the tangent of ∠ A (ratio of length of *Side a* to the length of *Side b*).

To get a better idea of how this works, consider this situation: A flagpole casts a shadow 10 meters long. The angle the shadow and the tip of the flagpole make with the ground is measured and is found to be 45°. What is the height of the flagpole?



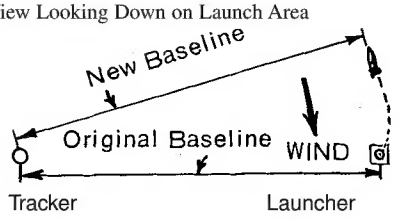
The flagpole in the picture appears to be about as long as its shadow. Checking the Tangent Table, the tangent of 45° is 1.00. Multiplying 10 meters times 1.00 gives a product of 10.0 meters. Our estimate turns out to have been accurate. A quick examination of the table shows that *rockets reaching angular distances of under 45° do not go as high as the baseline is long, but those going over 45° reach altitudes greater than the baseline's length.*

Two points to remember-

1. Rockets flown on windy days will usually not go straight up and will not go as high as they could have gone.
2. To minimize errors in altitude measurements for rockets going into the wind ("weather cock ing"), station the tracker at right angles to the wind flow.

The rocket moves into the wind, causing a slight increase in the length of the baseline. This introduces small error. The greater this "weathercocking", the greater the error. However, calculations are still based on the original measured baseline, so the altitude measurements computed will actually be a little low. Since every rocket launched from the launch pad will have approximately the same problem with the wind, reasonably accurate comparisons can be made between altitudes reached by different flights.

View Looking Down on Launch Area



If the tracker were stationed either upwind of the launcher (into the wind) or downwind of the launcher (away from the wind), the amount of change in length of baseline caused by weathercocking would be fairly great as compared to the change in baseline for a tracking station at right angles to the wind.

For more information on altitude tracking, refer to Estes Technical Report TR-3. Altitude Tracking in "The Classic Collection".

Reprinted from MRN Vol. 9, No. 3.

HOW FAST DID IT GO?

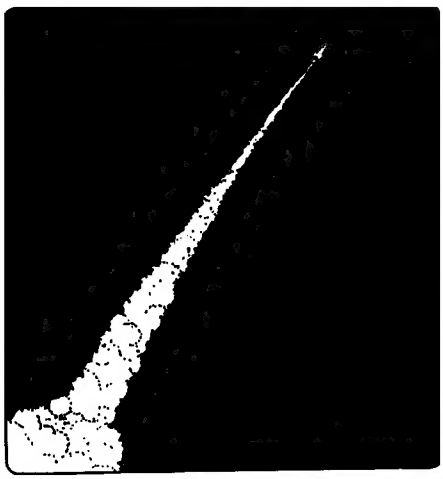
by Robert L. Cannon

A brief hiss of escaping gasses, a blur of motion and the rocket streaks skyward. Your model rocket really moves!

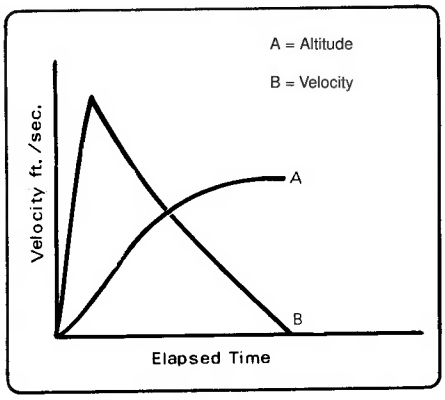
Have you ever stopped to wonder just how fast your model rocket travels? You have noticed that a large, relatively heavy rocket takes off fairly fast with a less-powerful engine, but doesn't rise too high before the propellant is gone. Then the delay and smoke tracking element of the engine starts to produce a trail of smoke so you can watch your rocket as it coasts upward. Soon it coasts to apogee, and if you selected an engine with the right delay, the parachute blossoms out just as it starts to tip over and begins to fall toward the ground.

If you observed carefully, you noticed that the rocket was gaining speed very quickly as it started on its flight into the sky. When you launched a small, relatively lightweight rocket with a powerful engine, you noticed that it took off very fast. Of course it went much higher than the heavy rocket with the less powerful engine.

In fact, you may have had the experience of launching a very small, light rocket with a powerful engine and actually loosing sight of it until the delay element began to leave a smoke trail high in the sky.



To produce enough thrust to move a tiny rocket, like a Mosquito™ with a C6-7 engine, to an altitude of 1700 feet in less than nine seconds (1.70 seconds of thrusting flight and 7 seconds of coasting flight), the small rocket's engine must cause the rocket to move very fast. An average speed for this upward flight would be 195.4 feet per second (1700 feet divided by 8.7 seconds).



Actually, the rocket moves faster and faster as the engine is thrusting. At the end of this thrusting portion of the flight (1.7 seconds into flight time from liftoff), the model rocket is traveling at its maximum speed. This maximum speed is 670 feet per second or about 3.5 times as fast as the average speed.

After the propellant is gone, the rocket is moving upward without a thrust force pushing it on up. The force of gravity acts to slow the rocket down.

Your rocket, while moving at its maximum velocity of 670 feet per second, is traveling very fast. To convert this speed into the speed you are familiar with estimate this speed as miles per hour. Put your answer in the space below.

670 feet per second = _____ m.p.h.

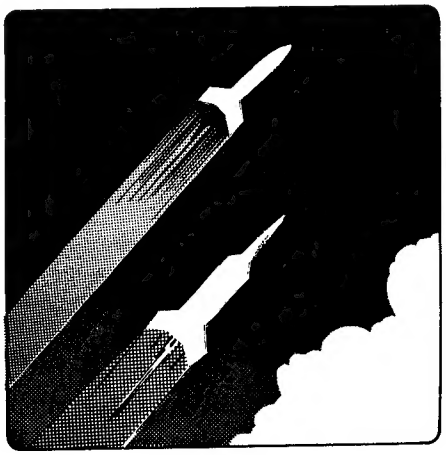
A speed of 670 feet per second is about 456 miles per hour. Your model rocket was really moving by the time the propellant was all gone.

When you fly a larger, much heavier model rocket with a smaller engine, as a Big Bertha with an A5-2 engine, it reaches a maximum velocity of 84 feet per second during its 2.8 second flight to parachute ejection. Multiply by the conversion factor of 0.68 to convert from feet per second to miles per hour. What was the maximum speed of this rocket in miles per hour?

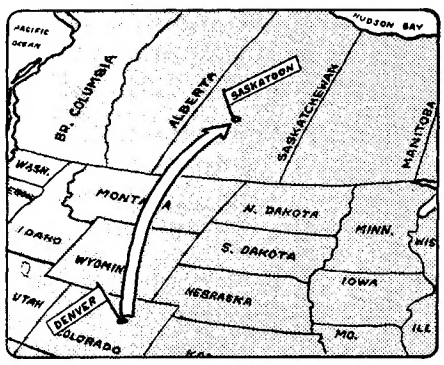
84 feet per second = about ___ m.p.h.

The rocket reached a maximum speed of about 57 miles per hour.

This speed is certainly not as fast as the 457 miles per hour which the other rocket reached. When you consider the fact this rocket with its engine weighed over 2.5 times as much as the other rocket (2.84 ounces as compared to 1.075 ounces), had an engine with one quarter the power (total impulse) of the other rocket's engine and had much greater drag, you should not be surprised that the heavy rocket only reached a speed of about one-eighth as great as that reached by the smaller rocket.



A jet plane traveling at the speed of sound near sea level goes about 750 miles per hour. This is about 1100 feet per second. A speed of 750 miles per hour is quite impressive. For example, a plane traveling from New York City to St. Louis, Missouri or from Denver, Colorado to Saskatoon, Saskatchewan, Canada in one hour.



We don't really think of this as being very far, but 750 miles (the distance a plane can go in one hour) is equal to going the distance all the way around a typical city block in 3,300 times in one hour.

A man walking or marching at a steady pace can average about three miles per hour. To convert this speed to feet per second, multiply the number of miles per hour by 1.47. Write this speed in feet per second in the space below.

3 miles per hour = _____ feet per second

A speed of three miles per hour equals 4.41 feet per second.

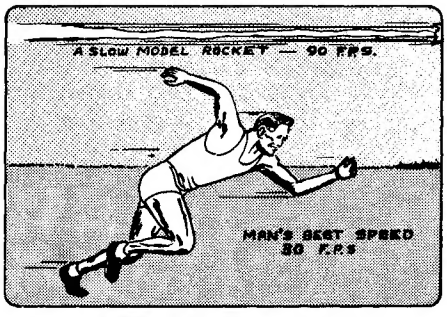
This speed isn't very fast, but it is in the range of speed with which you have personal contact. A very fast racer can run 100 yards in ten seconds. This is how many feet per second?

100 yards in 10 seconds = _____ feet per second

This speed is 30 feet per second (100 yards x 3 feet = 30 feet per second). This is a very fast speed for a man to move under his own power. How fast is this in miles per hour?

30 feet per second = _____ miles per hour

This speed is about 20 miles per hour. This is one of the fastest speeds a human can achieve on land through the use of his own muscles. One of the slowest model rocket launches produces a velocity which is nearly three times this fast.



Satellites traveling around the Earth vary in velocity from about 4.85 miles per second (at an altitude of 100 miles) to about 1.91 miles per second (in synchronous or 24-hour orbit). These speeds are so fast that it is difficult to realize how fast they really are. Remembering that a walking man can travel at a velocity of 4.41 feet per second, a fast runner can sprint (dash) at 30 feet per second for short periods, a model rocket can move at 670 feet per second, a jet traveling at the speed of sound goes 1100 feet per second, calculate the speed in feet per second for a satellite orbiting Earth at an altitude of 100 miles.

4.85 miles per second = _____ feet per second

To convert miles per second to miles per hour, multiply the speed in miles per second by 3600 (the number of seconds in an hour).

4.85 miles per second x 3600 seconds per hour = 17,460 miles per hour.

17,460 miles per hour x 1.47 = 25,666.2 feet per second.

This speed, about 25,700 feet per second, is over twenty-two times the speed of a jet plane flying at the speed of sound.

To attempt to give you a better understanding for these speeds, realize that it takes you about-

--68 seconds to walk the length of an average city block (300 ft.),

--10 seconds to run the same distance,

--0.45 seconds for a "hot" model rocket to go that far,

--0.27 seconds for a jet plane flying at the speed of sound or

--0.01 seconds for a satellite in an orbit of 100 miles altitude to go that far.

These numbers are given, not to make you feel that model rockets go slow (they don't), but to help you understand how their speeds compare to other velocities of objects with which you may someday work. If you have a strong desire and develop the necessary skills through study and careful practice, maybe you will someday be working with planes, full-scale rockets and satellites with velocities in this range. Model rocketry is one way to practice elements of some of the skills you will need to develop to become an aerospace scientist.

TABLE OF SPEEDS	
feet per second	Object
4.41 (walking 30 (short sprint)	A man
670	A model rocket
1100	A jet plane
25,666.2	Earth orbiting satellite (100 miles)

Reprinted from MRN Vol. 10, No. 1.

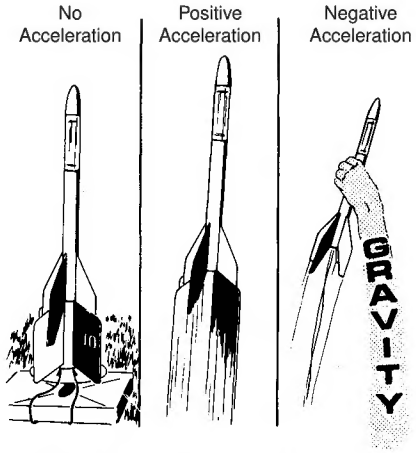
ACCELERATION AGAINST GRAVITY
OR
THE UPS AND DOWNS OF ROCKET FLIGHT

BY
ROBERT L. CANNON

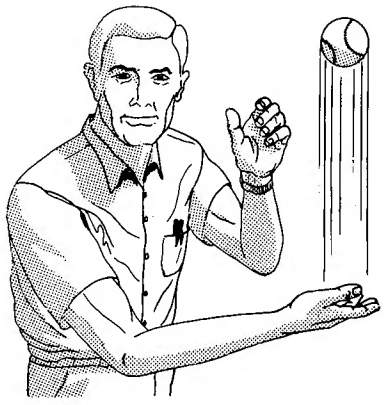
Acceleration is the process of speeding up. If something begins to go faster it accelerates.

Your model rocket sitting on the launch pad is not accelerating. When it starts to move, it accelerates. As long as its speed is increasing, the rocket is accelerating.

To be technical, an object which is gaining speed is undergoing positive acceleration. A moving body which is slowing down is undergoing negative acceleration. Negative acceleration is sometimes called deceleration.



A good example of acceleration occurs when you throw a ball straight up into the air. The ball is going very fast as it leaves your hand.



Once the ball leaves your hand it does not go any faster, in fact, the ball start slowing down as it rises.

What are the two forces acting on the ball to slow it down?

Two forces slowing down the ball:

1.

2.

The two forces are gravity and drag (friction between the ball and the air through which it traveling).

The ball soon loses all of its momentum, stops going up and starts to fall back to the ground. The rate at which the ball gains speed (acceleration) as it falls is about 32 feet per second per second (32 ft/sec^2).

This expression (32 feet per second per second) means that a falling object near the ground falls 32 feet per second faster for each second that it falls. In other words, a ball dropped from the top of a tall building will fall 16 feet during its first second of fall. Does this surprise you? Let's take an example to help us understand this.

If a car travels for one hour and begins its trip from a standing start and very slowly and steadily accelerates until it is traveling at a rate of 60 miles per hour, what was the car's average speed during this hour of travel?

$$\text{Average Speed} = \frac{\text{final speed + original speed}}{2}$$

Average Speed =

The car's average speed was $\frac{60 + 0 \text{ miles per hour}}{2}$ or 30 miles per hour.

A car traveling at the average rate of 30 miles per hour for one hour travels 30 miles in that hour. A ball accelerating at the rate of 32 feet per second travels 16 feet in the first second of its fall.

How far does a falling object travel during two seconds?

$$\text{Average Speed} = \frac{\text{final speed + original speed}}{2}$$
$$= \frac{\text{64 feet per second + 0}}{2}$$
$$= \text{32 feet per second}$$

Distance Traveled = Average speed x Time in motion

= 32 feet per second x 2 seconds

= 64 feet

How far did the falling ball travel during the second one-second of its fall?

Distance Traveled in the Second
Second = Total distance traveled - Distance traveled in first second

= 64 feet - 16 feet

= 48 feet

To simplify calculations we can use formulas instead of having to think through all of the steps in a problem each time we need to solve another problem of the same type. For example, the formula for determining the average speed of a body is--

$$\bar{v} = \frac{v_2 + v_1}{2}$$

$\bar{v}$ = average velocity

v_2 = final velocity (speed)

v_1 = original velocity

The formula for determining the distance an object falls during a given time is--

$$s = 1/2\ g\ t^2$$

s = distance

g = acceleration due to gravity

t = time

How far does a falling body travel during the third second of its fall?

$s = 1/2\ gt^2$

$s = 1/2 \times 32\ \text{feet/second}^2 \times (3\ \text{seconds})^2$

$= 1/2 \times 32\ \text{feet/second}^2 \times 9\ \text{seconds}^2$

$= 1/2 \times 32\ \text{feet} \times 9$

$= 16 \times 9\ \text{feet}$

$= 144\ \text{feet}$

The ball falls 144 feet in three seconds. Since the ball fell 64 feet in two seconds, the ball falls 80 feet during the third second of its fall (144 feet minus 64 feet).

A falling object keeps accelerating because of the force of gravity acting on it until the friction of the air moving past the falling body prevents the object from falling any faster. When this maximum speed is reached the object ceases to accelerate and falls at its terminal velocity.

LET'S TRY A PROBLEM

Using the following formulas (some are simplified to avoid using higher mathematics) we can determine some values for accelerations and velocities for model rockets. The values given are based on theoretical "no drag" conditions.

$$v_2 = \left(\frac{F}{w_{av}} - 1\right) gt$$

w_{av} = average weight of rocket
F = force (average thrust of rocket engine)
g = acceleration due to gravity (32 ft./sec.^{2})
t = time in seconds

ALPHA FLIGHT ANALYSIS

Using the above formula for velocity, determine the burnout velocity of an Alpha launched using an A8-3 engine. The Alpha weighs 0.8 ounces without an engine. With an A8-3 engine the Alpha weighs 1.37 ounces at liftoff. The weight of propellant in an A8-3 engine is 0.11 ounces giving an average weight of 1.32 ounces during the thrust phase of the flight. The A8-3 engine thrusts for 0.32 seconds and has a total impulse of 0.56 pound-seconds. (These values may be found in or calculated from information in the current Estes catalog.)

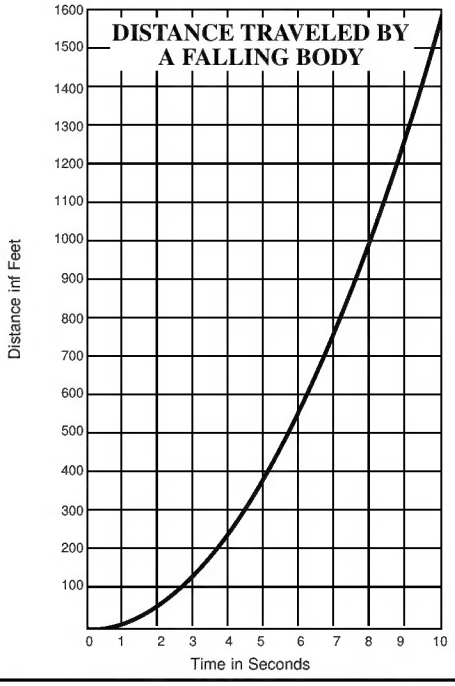
First find the average force, then use this force in the velocity formula to find the final velocity.

$$\text{Force (thrust)} = \frac{\text{total impulse}}{\text{burn time}} \times \left\{\frac{16\ \text{oz.}}{1\ \text{lb.}}\right\}$$

conversion factor

A graph can present a lot of information quickly. Studying a graph can sometimes help you to understand something that may be hard to understand otherwise.

Examine the graph for the distances a falling body travels during the first few seconds of its fall. This graph represents the distance a body falls under the constant acceleration of gravity (neglecting air friction).



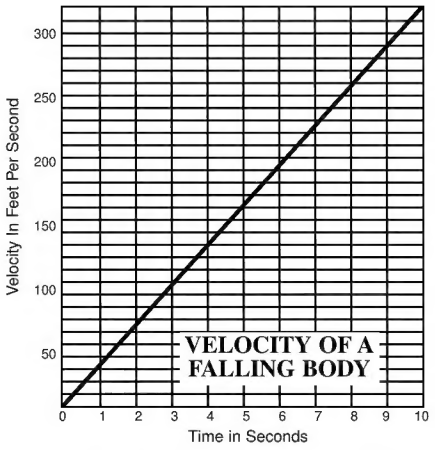
GRAPHS

To help understand the velocities which falling objects can develop, (neglecting air friction) examine a graph of the velocities developed by freely falling objects. These graphs are based on results obtained by use of these formulas-

$v_2 = a\ t$

$s = v_1\ t + \frac{at^2}{2}$

a = acceleration



To find the actual velocities which your birds will attain when allowance for air drag is made, use the Estes book "Altitude Prediction Charts" (EST 2842).

$$F = \frac{0.56\ \text{lb.} - \text{sec.}}{0.32\ \text{sec.}} \times \frac{16\ \text{oz.}}{1\ \text{lb.}}$$

$= 1.75 \times 16\ \text{oz.}$

$= 28.0\ \text{oz.}$

$$v_2 = \left(\frac{F}{w_{av}} - 1\right) gt$$

$$v_2 = \left(\frac{28.0\ \text{oz.}}{1.32\ \text{oz.}} - 1\right) (32\ \text{ft./sec.}^2)(0.32\ \text{sec.})$$

$= (21.21-1)(32\ \text{ft./sec.}^2)(0.32\ \text{sec.})$

$= (20.21)\ (10.24\ \text{ft./sec.})$

$= 206.95\ \text{ft./sec.}$

The expression $\left(\frac{F}{w_{av}} - 1\right)$ gives you an idea of the number of "gravities" the rocket experiences in upward flight during acceleration. The "1" is subtracted in this expression to allow for the pull of Earth's gravity on the rocket. This rocket develops a fairly high velocity by the end of the thrusting phase of flight.

What will be the velocity developed by the same Alpha launched with a C6-5 engine?

$$F = \frac{2.25\ \text{lb./sec.}}{1.70\ \text{sec.}} \times \frac{16\ \text{oz.}}{1\ \text{lb.}}$$

$=$

$=$

$=$

$$v_2 = \left(\frac{}{1.49\ \text{oz.}} - 1\right) (32\ \text{ft./sec.}^2)\ (1.7\text{sec.})$$

$=$

$=$

$=$ ft./sec.

This velocity (716.45 feet per second) developed by the Alpha with the C6-5 engine is more than triple the velocity (206.95 feet per second) which was developed by the A8-3 engine. Notice that the thrust and therefore the acceleration in "g"s produced by the C6-5 engine (13.17 g) is less than that produced by the A8-3 engine (20.21 g), but the maximum velocity is greater when using the C6-5 engine because the burn time for this engine is greater.

I wonder what the acceleration would be in "g"s for the Big Daddy™ with a D12-5 engine? Hmm...

Reprinted from MRN Vol. 10, No. 2.